Exercise 3 – 03.10.2024

Perfect plasticity

1. Parameter determination for two yield criteria

Drained triaxial tests were conducted on a silty sand under various stress paths. The initial pore water pressure was zero. Experimental results are available for CTC (Conventional Triaxial Compression), RTE (Reduced Triaxial Extention), CTE (Conventional triaxial extension), TC (Triaxial Compression), TE (Triaxial Extension) tests, as displayed in annex 2.[1] It is required to find the parameters for the Mohr-Coulomb and Drucker-Prager failure criteria.

Part 1

Consider the two CTC tests (figures 1 and 2) and the RTE test (figure 3). For each test:

- Compute the values of σ_1 and σ_3 at *ultimate state*[2] and the invariants $\sqrt{J_{2D}}$ and J_1
- Plot the obtained results[3] in a plane of your choice (σ_m -τ_{max}), (σ - τ) or (p'-q) to obtain the parameters (c', φ') of the Mohr – Coulomb criterion in compression.
- Plot the obtained results in the plane (J_1 – $\sqrt{J_{2D}}$) to obtain the parameters (α, k) of the Drucker-Prager failure criterion.

Part 2

Combine the previous results with the RTE test (figure 4), the CTE test (figure 5), the TC test (figure 6) and the TE test (figure 7).

For each test:

- Compute the values of σ_1 and σ_3 at *ultimate state*[2] and the invariants $\sqrt{J_{2D}}$ and J_1
- Plot the obtained results in a plane of your choice[3] (σ_m -τ_{max}), (σ - τ), (p'-q) to find the parameters (c', φ') of the Mohr – Coulomb criterion in compression and extension.
- Plot the obtained results in the plane (J_1 - $\sqrt{J_{2D}}$) and find the parameters (α, k) of the Drucker-Prager failure criterion.

[1] The results have been taken from (Desai C.S. and Siriwardane H.J., Constitutive Laws for Engineering Materials with Emphasis on Geologic Materials, Prentice-Hall, 1984) changing the unit system.
[2] For the present case, where perfect plasticity is assumed, the ultimate state is defined as the point where the stress reaches its maximum value.
[3] We use the conventional notation for axes: 'x-coordinate versus y-coordinate'.

Appendices

Annex 1

Undrained shear modulus G from deviatoric stress and invariants:

- The deviatoric stress q is related to the invariant I_{2D} by the shear modulus G, according to the following equation:

$$q = 2\sqrt{3}G\sqrt{I_{2D}}$$

- This equation is obtained from the relationship between deviatoric stress q and the second invariant J_{2D} of the deviatoric stress tensor s_{ij}:

$$J_{2D} = \frac{1}{6}[(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2] = \frac{1}{6}[2(\sigma_1 - \sigma_3)^2] = \frac{1}{3}[(\sigma_a - \sigma_r)^2]$$
$$\mathrm{q} = \sqrt{3\mathrm{J}_{2\mathrm{D}}}$$

- Considering the elastic constitutive relationship between deviatoric stress tensor s_{ij} and deviatoric strain tensor e_{ij}:

$$s_{ij} = 2Ge_{ij}$$

- J_{2D} can be written as:

$$\sqrt{\mathrm{J}_{2\mathrm{D}}} = 2\mathrm{G}\sqrt{\mathrm{I}_{2\mathrm{D}}}$$

where, in triaxial conditions, I_{2D} is second invariant of the deviatoric strain tensor e_{ij} :

$$\mathrm{I}_{2\mathrm{D}} = \frac{1}{6}[(\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2] = \cdots = \frac{1}{3}(\varepsilon_a - \varepsilon_\mathrm{r})^2$$

- Finally, we can write $q = 2G(\varepsilon_{11} - \varepsilon_{33})$, and $\Delta q = 2G(\Delta\varepsilon_{11} - \Delta\varepsilon_{33})$

Annex 2

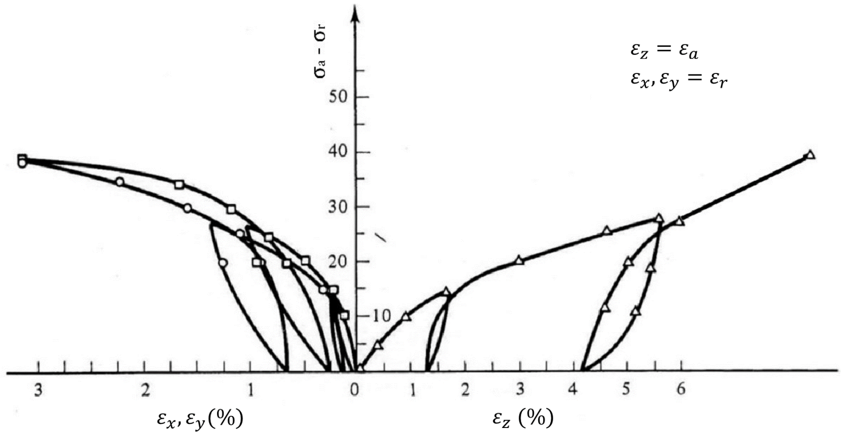

Figure 1. Stress-strain curves from CTC conventional triaxial compression stress path (stresses are in kPa). Radial stress is kept constant at σ_r=10 kPa.

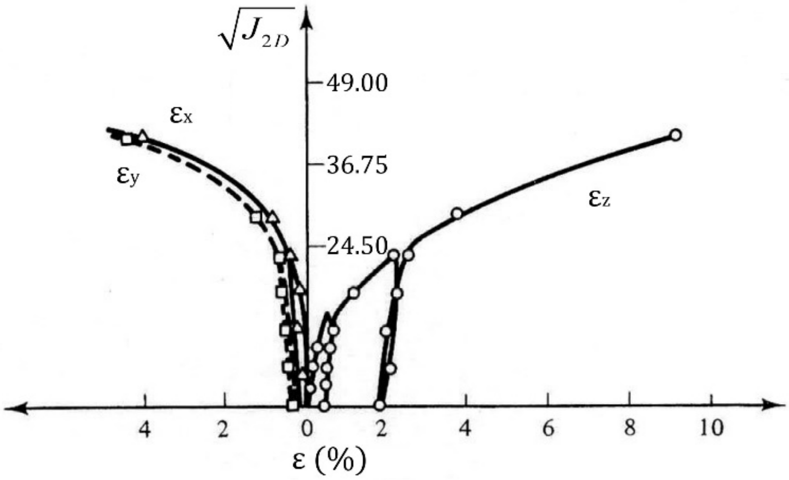

Figure 2. Stress-strain curves from CTC conventional triaxial compression stress path (stresses are in kPa). Radial stress is kept constant at σ_r=20 kPa.

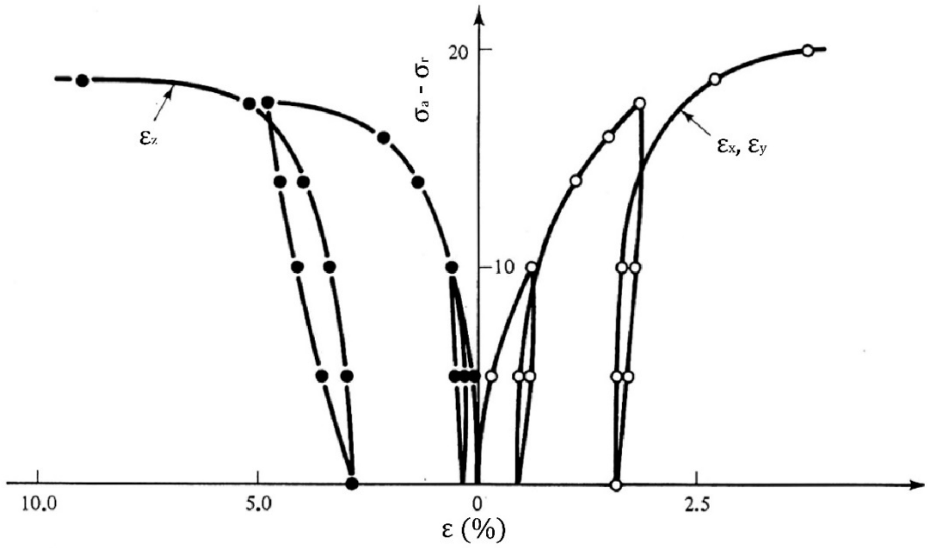

Figure 3. Stress-strain curves from RTE stress path (stresses are in kPa). Radial stress is kept constant at σ_r=20 kPa. Note that the values on the y-axis are negative.

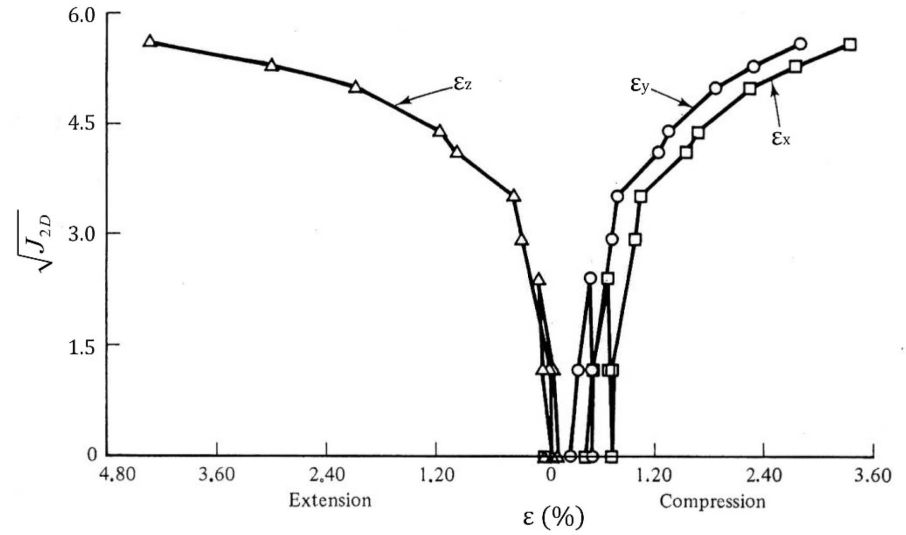

Figure 4. Stress-strain curves from RTE stress path, (stresses in kPa). Radial stress is kept constant at σ_r=10 kPa.

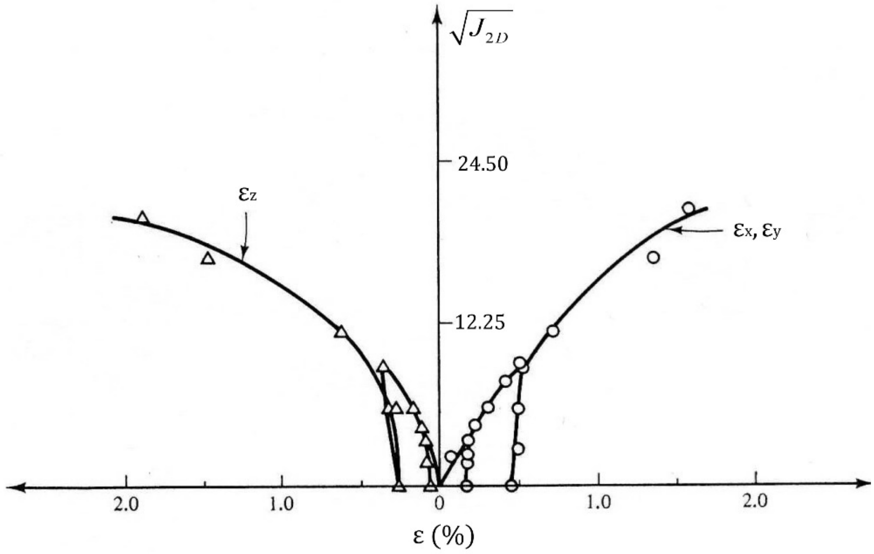

Figure 5. Stress-strain curves from CTE stress path, (stresses in kPa). Axial stress is kept constant at σ_a=20 kPa.

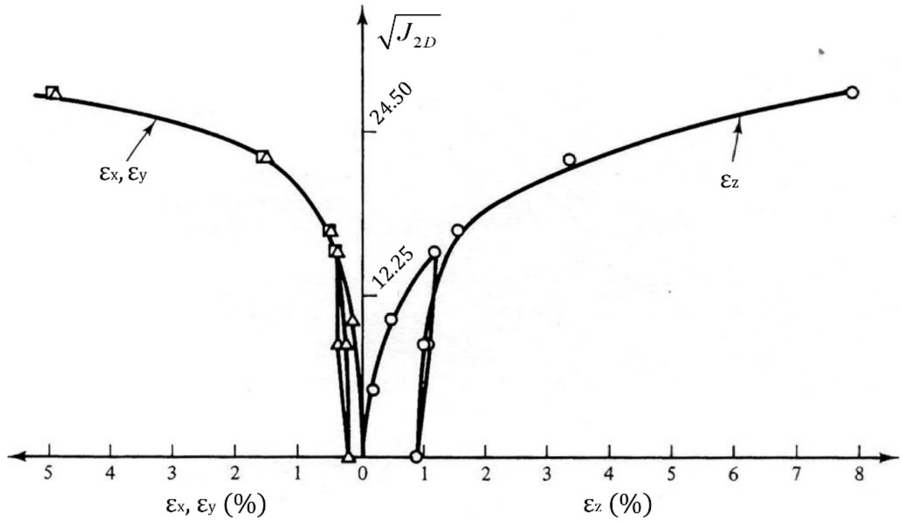

Figure 6. Stress-strain curves from TC stress path (stresses are in kPa). Initial radial stress is σ_r=25 kPa.

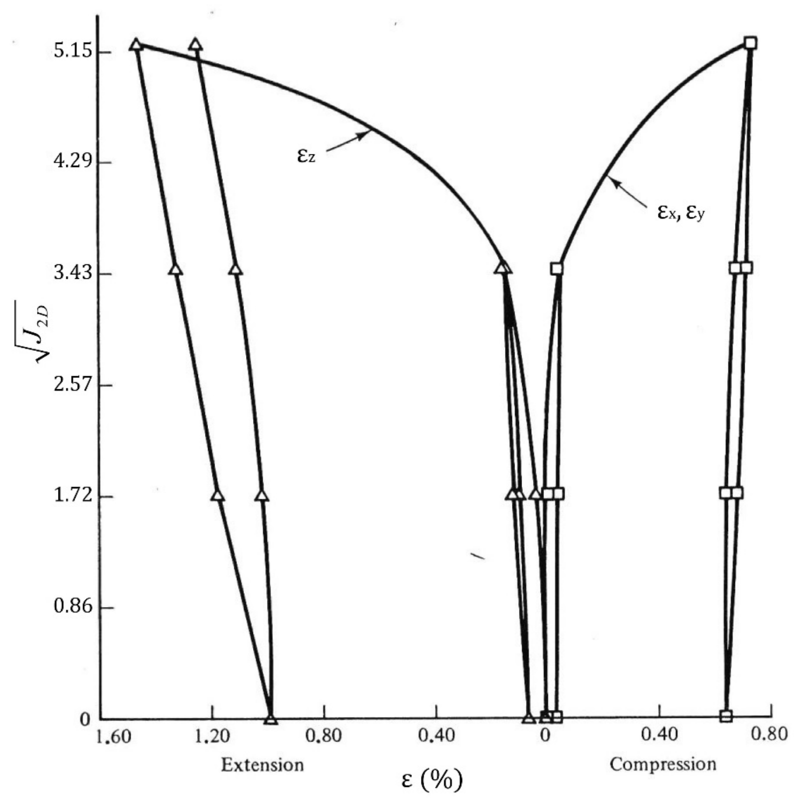

Figure 7. Stress-strain curves from TE stress path, (stresses are in kPa). Initial radial stress is σ_r=10 kPa.